The Cost of Capital in Regulated Firms: The Argentine Experience
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Abstract:

The sustainability of service is one of the fundamental aims of any regulatory system, and this tends to be reflected in the general principles within the legal framework of each public service. The necessity to cover economic costs of service then follows, in order to cover costs in such a way as to guarantee the maintaining and continuity of the system as well as its expansion. These economic costs should necessarily include sufficient retribution for the capital used by investors: the cost of capital.

In the case of Argentine regulation, it is usual practice to use an adapted form of the CAPM (Capital Asset Pricing Model) to calculate the opportunity cost of equity, in such a way as to take the country risk into account. In this work we will analyse the components of the adapted formula for the cost of capital, discussing incentives which generate each of the possible outcomes for the regulated firms. Finally, we will present a model of dynamic optimisation for a regulated firm in a country such as Argentina.
1. Introduction

The sustainability of service is one of the fundamental aims of any regulatory system and, in Argentina, this tends to be reflected in the general principles within the legal framework of each public service. In Argentina, in the case of water, this aim of sustainability is included in the Regulatory Structure (Decree 999/92), both in the general objectives (Article 3) and in the tariff principles. Similar concepts can be found in the law of the regulatory framework of electricity (Law 24065) and gas (Law 24076). The necessity to cover economic costs of service then follows, in order to cover costs in such a way as to guarantee the maintaining and continuity of the system as well as its expansion. These economic costs must necessarily include sufficient retribution for the capital used by investors.

In competitive markets, the payment for capital is manifested as a result of the exploitation of service, as long as investors are residual recipients of profits. That is, investors receive the surplus income once the costs of providing the good or service have been covered. The very mechanism of competition guarantees that these profits will be reasonable every time an increase in the rate of return of a sector attracts capital while simultaneously increasing supply and reducing prices and profits to their equilibrium level.

In those cases where there is a natural monopoly, this automatic mechanism does not function since the monopoly-holder can increase its rate of return by increasing prices and restricting supply, with no risk of attracting new entries into the market. Given the negative consequences this may have for social welfare (which go beyond mere problems of distribution) we can see the emergence of the need for some regulation of monopolies.

The aim of regulation can be seen in this context as being a substitute for competition, in terms of inducing the sort of behaviour in a firm that will lead to a maximisation of social welfare. This implies the establishment of price and service conditions such that investors reach a rate of return which is similar to that which they would derive from a competitive market: this should be sufficiently high as to attract the capital necessary for the activity without allowing the excessive profits resulting from the monopoly power.

The regulated variable is therefore the rate of return obtained by the monopoly-holder, with this regulation being direct (regulation via the rate of return or the cost of the service) or indirect (through price cap regulations). Although these two mechanisms differ in a number of ways (particularly in terms of the incentives to productive efficiency which they offer) these differences are matters of degree rather than of principle, and in both cases the guaranteeing of the sustainability of service appears as a key element.
As has already been stated, sustainability is intimately related to the possibility of investors recovering invested capital and obtaining fair and reasonable reward as compensation for their capital invested. American doctrine is important here, corresponding as it does to the country with the greatest tradition of regulation.

The criteria for determining the justice and reasonableness of a rate of return were fixed by the United States Supreme Court in two judgements which established the prevailing doctrine on the subject, namely:

*Bluefield Water Works & Improvement Co. v. Public Sector Commission of West Virginia* (262 U.S. 679, 1923)


These cases established the following criteria for determining the justice and reasonableness of the rate of returns:

a) a standard for the attraction of capital

b) a standard for comparable profits

c) financial integrity

According to Morin (1994) the economic logic of these criteria is direct in terms of the existence of an opportunity cost of capital brought by the investors to regulated firms. This opportunity cost is given by the expected value of profits which would have been obtained from the best available investment alternative among similar-risk firms.

Additional refinements to these judgements were established by the United States Supreme Court, recognising the difference between regulatory risks (*Duquense Light Company et al. v. David Barasch et al.*) and principles of capital attraction (*British Columbia Electric Railway v. Public Utilities Commission of British Columbia et al.*).

Another important element arising from the Hope case is called the “end result” doctrine, which states that the regulatory methods used are immaterial, provided that the final result is a reasonable one for both investor and consumer. In other words, regulators are not obliged to use any particular technique, so long as the results obtained are fair and reasonable. Any inherent weaknesses found in the different methods therefore become of secondary importance.

Criteria similar to those of the Hope and Bluefield doctrine have been incorporated into Argentine legislation. These principles have been generally manifested in most regulatory frameworks existing in the country. Taking Law 24065 as an example, that of the Regulatory Structure of the Argentine Electricity Sector, when referring to the profitability of regulated firms the following principle is established:

*ARTICLE 41 - the tariffs for transporters and distributors should allow for a reasonable rate of return for those companies operating efficiently. Likewise, the rate should: a) maintain some relation to the degree of efficiency and operative effectiveness of the firm; b)*
be similar (as an industry average) to that of other national or international activities of similar or comparable risk.

The Tariff Principles of the Regulatory Structure of Water, again in Argentina, include among their general principles:

1) Prices and tariffs would tend to reflect the economic cost of the provision of the service including the profit margin of the producer and incorporating those costs emerging from approved expansion plans.

Although in this case there are no explicit definitions for the “profit margin”, the criteria of sustainability set up in advance would necessarily lead to a need to meet standards for capital attraction, comparable profits and financial integrity as set up in American doctrine.

Before embarking fully on a determination of the cost of capital, we ought to stress that there is a need for consistency within the regulatory system as a whole, making it impossible to determine the cost of capital in a way that is isolated from the group of norms already in place and the way in which these norms are interpreted and applied. Far from being independent elements which figure in the firm’s economic equation, the distinct elements are intimately interrelated and the accurate determination of each depends closely on assumptions and methods used in the determination and evaluation of the others.

By way of example, we might consider the interrelation between the determination of the cost of capital and that of the asset base on which it will be applied. Among the forms of asset determination we can use a historical valuation of capital, or alternatively, a valuation based on replacement values. The method chosen has a direct impact on the cost of capital, as a valuation based on replacement values is more risky for shareholders than a historical one. Gordon (1977) shows how adopting reinvestment values can result in an increased cost of capital reflecting this greater risk.

In general, this work will discuss issues related to the cost of capital in regulated firms, with an emphasis on Argentina. The structure of the work is as follows: Section II will present the formula for the cost of capital and will discuss its components; Section III will discuss the relation between the cost of capital and the firms’ incentives, with an emphasis on those variables over which the firm has direct control (the cost of debt and the capital structure); the following section analyses the relation between the risk faced by the regulated firm and the kind of regulation to which it is subject; in Section V we will discuss the adaptation of the CAPM to the Argentine reality, while in Section VI we will develop a model of optimum control, showing how a regulatory system and the clauses of the concession contract determine investment policy. The model is geared towards showing those aspects related to the case of Argentina. Finally, in Section VII we shall offer our conclusions.

II. Measuring the Cost of Capital

In general, firms are financed with two kinds of funding: equity and debt. Debt and equity differ in that creditors have priority rights over the firm’s profits, while with equity only residual
rights exist (i.e. they receive whatever is left over after the creditors are paid). We should not be surprised that equity is considered more risky and, therefore, that the cost of equity is higher than the cost of debt.

The cost of capital is an average derived from the cost of debt and the cost of equity. Formally, this is

$$r_k = r_d (1-t) \frac{D}{V} + r_{kp} \frac{KP}{V}$$

where $r_k$ is the cost of capital of the firm after tax, $t$ is the tax rate on earnings paid by firms, $r_d$ is the firm’s cost of debt, $r_{kp}$ the cost of equity, $KP$ the value of equity, $D$ the value of debt and $V$ the total value of assets or the total capital of the firm ($V = D + KP$). The $D/V$ relation is called financial leverage, with $r_d(1-t)$ being the cost of debt after tax.

**Weightings**

There are basically two options for obtaining the level of leverage and the participation of equity in total capital: book value and market value. The advantage of book value is that it involves a figure which is stable over time and is available for all companies. However, when a historical system of accounting is used, the true value of the firm is given more precisely by the market value. Furthermore, depreciatory systems vary from country to country, which affects the book value given by accounts, and makes it difficult to draw comparisons between firms of different nationalities. The market value, in turn, has the significant disadvantage that the majority of firms are not quoted on the stock exchange, and so their market values are not available. Even for those firms which are quoted on the stock exchange, the majority of their debt is not (Alexander, 1995). There is, furthermore, a problem of circularity in the market value, as it is equal to the discounted cash-flow at an adequate discount rate, i.e. the opportunity cost of capital. That is, the market value will depend on market expectations of the opportunity cost of capital, which, in turn, helps to determine the future flows of funds.

In the context of determining the cost of capital of regulated firms, usual practice consists in calculating the cost of equity and that of debt for its respective book values (see Morin, 1994, p.411 or Spiegel and Spulber, 1994, p.426).

**The cost of debt**

The cost of debt is equal to the medium- and long-term cost of debt incurred by the firm to obtain funds to finance its projects. In principle there are two options, which ought to be considered: the average cost and the marginal cost of debt.

The average cost of debt can be found by dividing the interest paid (plus costs of issuing of the debt) by the book value of the debt. This result provides the rate that the firm is actually paying for the debt already contracted. Allowing the cost of debt to be the real cost of the debt, and not the marginal market cost (the marginal cost of acquiring a debt) prevents shareholders from acquiring unexpected losses or profits in the case of fluctuations in the interest rate. The idea of those who support the use of average cost is that interest payments the firm must meet do not depend on the market’s current interest rate but on the rate which was in effect at the time when obligations were undertaken. If one expects future interest rates to be different from those the
firm is actually paying, and the company had planned to take on more debt in the future, then these differential costs should be incorporated, as far as possible, into the calculation of \( r_d \).

The debt considered should be such that its period of expiry is comparable to the period being analysed. Discussion on this point centres on whether the period considered is that up to the ending of the concession or that up until the next tariff review. A part of the answer depends on whether or not the firm has the right to leave the agreement at the moment of the tariff reviews.

In general, the cost of debt is determined by the following variables:

- The level of the interest rate. An increase in the interest rate leads to an increase in firms’ cost of debt.
- The risk of the firm defaulting. The greater the risk that the firm will not service its debts, the greater the cost associated with debt. Given that firms only respond to their creditors with their own capital, the D/KP relation is a good indicator of the risk of default. Formally, this is

\[ r_d = F(D/KP), \quad F' > 0 \]

where \( F' \) is that derived from function \( F \) with respect to the D/KP relation. That is, the cost of debt depends positively on the firm’s financial leverage.

- Tax advantages linked to debt. In many countries, including Argentina, interest paid is tax-deductible. This is why the post-tax cost of debt is a function of the tax rate. Tax profits from payment of interest leads to the post-tax cost of debt becoming lower than the pre-tax cost, with this profit increasing at the rate at which the taxation rate increases.

- Duration. The longer the duration, the greater the rate which will have to be paid in order to achieve financing.

The cost of equity

Theoretically there are a number of approximations to the problem estimating the cost of equity: the Capital Asset Pricing Model (CAPM), the Dividend Growth Model and the Arbitrage Pricing Model. The CAPM is used extensively on both sides of the Atlantic for estimating the cost of capital of both regulated and unregulated firms, and it is by far the dominant model (Grout, 1992). This does not mean that the CAPM gives a good prediction of the true opportunity cost of equity, it is simply the best of the known options.

**Capital Asset Pricing Model.** The thinking behind the CAPM is the following: the risk linked to the holding of a share derives from the uncertainty governing the returns on this share. Changes in the return on a share can be divided into two groups: those related to movements in the market within its group (systemic risk), and those which are not (specific risk). An investor does not require any return for taking a non-systemic risk, as he himself could protect himself from this risk by a suitable diversification of his portfolio. Therefore the contribution of the asset to the variability of the market portfolio (represented by beta) determines the rate of returns the share must pay.
One interesting aspect of this is the question of which non-diversifiable risks an investor will be faced with? The CAPM presupposes that the proportion of wealth invested in a share is small. This condition may be difficult to fulfill if the buyer of the firm is obliged to get a large percentage of it. The capacity for diversification is an empirical problem, since in spite of the large size of public firms, they can represent only a small part of the buyer’s portfolio.

The CAPM shows that the cost of equity of a share is equal to the return on a risk-free share plus the premium for the risk taken, which is equal to the value of beta (the covariance of the return on the share and the market’s return, divided by the variance of the market’s return) multiplied by the risk premium of the market. Formally, this is

\[ r_{kp} = r_1 + \beta (r_m - r_f) \]  

(1)

where \( r_m \) is the return on a diversified share portfolio and \( r_f \) is the risk-free rate.

It is important to emphasise the two fundamental assumptions behind the CAPM. The first is that capital markets are competitive and efficient, and that information is freely available to all agents. A market is efficient when its prices comprehend all the available information. This information is instantly absorbed by prices and, therefore, the current price of a share represents the best estimate of its true value. Empirical evidence is not conclusive when it comes to assessing the validity or otherwise of this assumption. The second general assumption is that investors are rational, and that their objective is to maximise profits, and in this maximisation process they will demand greater returns for running greater risk.

To apply the CAPM it is necessary to estimate parameters for (1) and then plug them into the CAPM formula in order to obtain an estimate for the cost of equity. Although the CAPM formula appears very simple, carrying out an estimation in practice presents a number of difficulties. The main difficulty is due to the fact that the CAPM is a prospective model, while most data necessary for calculating the three theoretical parameters (expected risk-free rate, expected beta and expected market return) are historical. Below we will discuss the practical problems related to the estimation of each of the parameters in (1).

The **risk-free rate** is the return on a asset with a default risk of zero. In developed countries the Treasury Bond rate is a proxy variable of the risk-free rate. Empirical studies usually use as their ‘risk-free rate’ the internal rate of return of a bond issued by the U.S. government, as it is presumed that agents consider the possibility of this government failing to honour its debts to be zero. The chosen bond should have a maturity similar to the firm’s life-span, or that of the share being evaluated. It is, however, worthwhile stressing that the prices of these bonds are only guaranteed if they are retained until their expiry. Furthermore, although nominal return is known for certain if a bond is retained until its expiry, the rate of inflation is uncertain and therefore its real rate of return of not free from risk (Berndt, 1993). In some countries (such as the U.K.) there are indexed bonds and these can therefore be used for determining the real interest rate.

The **market’s risk premium** measures the additional yield which an investor requires in order to maintain a diversified portfolio of shares instead of a risk-free asset. Some authors calculate the components of the risk premium separately, though common practice is to calculate the market’s risk premium directly, estimating it as an average of the past returns of an appropriate market index. The idea behind realising an average of past returns as a way of obtaining the
market average risk premium (MARP) presupposes that all historical returns have equal probability of occurring in the future. There are a number of aspects which have to be discussed at the moment of estimating the MARP:

- **Definition of the Market.** According to the CAPM, the market portfolio should include *all* risky assets, while the market indices used for estimating $r_m$ contain only a limited sample of shares. Increased transaction costs or the presence of legal regulations prevent certain assets being saleable, meaning that no portfolio could ever contain all the assets in the economy. Human capital is the most important example of this kind of assets.

- **Period of time.** As can be seen in the following Table, the estimate of MARP is very sensitive to the period in which the average is being calculated.

### TABLE 1
**Geometric and Arithmetic Averages of the spread between return on the S&P 500 index and the IRR of the 30-year bond of the U.S. government**

<table>
<thead>
<tr>
<th>Period</th>
<th>Arithmetic</th>
<th>Geometric</th>
</tr>
</thead>
<tbody>
<tr>
<td>1926-97</td>
<td>8.97%</td>
<td>7.14%</td>
</tr>
<tr>
<td>1946-97</td>
<td>7.65%</td>
<td>6.34%</td>
</tr>
<tr>
<td>1960-97</td>
<td>5.40%</td>
<td>4.22%</td>
</tr>
<tr>
<td>1970-97</td>
<td>5.83%</td>
<td>4.56%</td>
</tr>
<tr>
<td>1980-97</td>
<td>8.94%</td>
<td>8.09%</td>
</tr>
<tr>
<td>1990-97</td>
<td>10.28%</td>
<td>9.30%</td>
</tr>
</tbody>
</table>

*Source: Ibbotson Associates.*

In the periods chosen, the MARP fluctuates between 10.28% and 5.40% if an arithmetic average is used, and between 9.30% and 4.22% with a geometric one. That is, the empirical estimate of the cost of capital depends dramatically on the sample period used for calculating the MARP.
In general, the approach used for estimating the MARP is an historical one: it is assumed that the average of past returns is a suitable predictor of expected returns. In accordance with such an approach, the calculation of market risk should use the greatest period of time possible for which data are available. The idea is as follows: there are short periods in which investors earn more than expected, which are then compensated for by short periods in which they earn less than expected. It is only over long periods that expected and actual returns converge. Risk premiums based on short periods of time can be excessively volatile, dependent on circumstantial market factors.

- **The way of calculating averages.** It is possible to deduce from the above table that the choice of the way of calculating an average also affects the estimate for MARP. There are basically two alternatives: the arithmetic average and the geometric average. Carrying out a comparison of the two, one finds that a arithmetic average of a series of past returns is easier to calculate, but is biased upwards. This bias is greater, the greater the variance of returns. The geometric average, on the other hand, involves no bias and represents the true average return obtained by investors over a given period of time. However, in order to calculate the premium of average risk of the market, what interests us is not the average return on a past portfolio but the average return on a future one. By definition the cost of capital is the annual discount rate making the present value of a cash-flow equal to the firm’s current market price. Future returns are unknown, they are subject to chance, and to obtain the mean value of a random variable it is the arithmetic average which is the correct one to use (for an example, in clarification, see Morin, 1994). Synthesising this, we can see that the geometric average reflects the historical return which is actually obtained on a portfolio and therefore when looking at the past the geometric average is relevant. However, when it is a case of predicting future returns, the arithmetic mean is the unbiased measure of expected value of a repeated number of observations of an random variable. The geometric average, in turn, is biased downwards (that is, geometric returns produce values lower than arithmetic returns) due to the concave nature of the square-root function. It is, however, worth making clear that the above analysis assumes that returns are independent of each other. Although empirical studies support the validity of this assumption in the short term, in the long term there seems to be a process of reversion of the mean which invalidates it. This argument is used in support of those who defend the use of the geometric return. However, one consequence of challenging the use of the arithmetic mean, arguing that there is a process of reversion of the mean, is that this very argument implies that markets are not efficient (this knowledge could be used to find surplus returns); this refutes the use of the CAPM, since this model assumes that markets are efficient.

The **beta** of a share is the specific risk coefficient, calculated as the ratio between the covariance of the return of the company’s and market’s shares and the variance of the market’s returns. Formally, this is

\[
\beta_j = \frac{\sigma_{jm}}{\sigma^2_m}
\]

where \(\beta_j\) is the beta of the asset of firm \(j\), \(\sigma_{jm}\) is the covariance between the yield of the industry to which the firm belongs and the yield of the market, while \(\sigma^2_m\) is the variance of the market yield.
Beta can also be calculated econometrically. This process consists in carrying out a regression of ordinary least squares where the return (yearly, monthly, weekly or daily) on a share is the dependent variable, and a constant and the return (yearly, monthly, weekly, or daily) of an adequate index of the market are independent variables:

\[
R_{j,t} = \alpha_j + \beta_j r_{m,t} + \epsilon_{j,t}
\]

In the above equation, \( R_{j,t} \) is the return on share \( j \) at moment \( t \) (including capital gains) and \( \epsilon \) is the error term. Beta is simply the tangent of the regression line.

It is worth emphasising that the true value of beta for a firm can never be observed. Historical estimates for betas can only be used as proxies for the true beta, as the future can well differ from the past. For the model to be workable, the parameters of the model (particularly the beta coefficient) must be reasonably stable. Pettway (1977) analyses the stability of the parameters of the model for the U.S. electrical industry between January 1971 and December 1976, finding an initial long period of stability between 1971 and 1974, followed by a transitory period of instability, following which values of the parameters returned to their previous levels.

Estimates for absolute betas can differ for a number of reasons: the time period used, the choice of market index, and whether returns are annual, monthly, weekly or daily, are all factors which will influence the final result. In practice, the most-used time period is five years (Value Line calculated betas based on weekly returns over a five-week period, while Merrill Lynch did it with monthly returns, but also using a five-week period). There is extensive documentation in the literature of this phenomenon of betas changing as the interval at which returns are calculated varies. These changes are generally attributed to the infrequent commercialisation of papers, which contribute to the existence of differences between actual returns and those observed (Handa, Kothari and Wasley, 1989). Empirical results show that the betas of shares which are more risky than the market increase with the interval of returns, while betas of shares less risky than the market decrease with the interval of returns.

For those shares quoted on the stock exchange only infrequently, estimates of betas are biased downwards. Intuition suggests that this is due to the fact that the price of a share is often not modified simply because it is not quoted, which tends to diminish its covariance with the movements of the market. There are two approaches for adjusting betas for firms which are quoted infrequently, Dimson’s (1979) adjustment, and Scholes and Williams’ (1977).

Dimson’s adjustment is carried out by running a regression on ordinary least squares (OLS) of the return on a share against the return on the market, and the return on the market in the preceding period, and then adding the slope parameters.

\[
R_{j,t} = \alpha_j + \beta_{j,1} r_{m,t} + \beta_{j,2} r_{m,t-1} + \epsilon_{j,t}
\]

\[
\beta = \beta_{j,1} + \beta_{j,2}
\]

where \( r_{m,t-1} \) is the market return for the preceding period.

In the approach proposed by Scholes and Williams, three regressions are estimated for OLS between the return on the share and the market return, the market return for the preceding period
and the market return for the subsequent period. The corrected beta coefficient is obtained by adding the three slope parameters, and then dividing the result by one plus twice the coefficient of autocorrelation of the market returns over time. Formally this can be written as

\[
R_{jt} = \alpha_j + \beta_{j,1} r_{mt} + \varepsilon_{jt}
\]

\[
R_{jt} = \alpha_j + \beta_{j,2} r_{mt-1} + \varepsilon_{jt}
\]

\[
R_{jt} = \alpha_j + \beta_{j,3} r_{m,t+1} + \varepsilon_{jt}
\]

\[
\beta = (\beta_{j,1} + \beta_{j,2} + \beta_{j,3})/(1+2\rho)
\]

where \(\rho\) is the coefficient of autocorrelation of market returns over time, and \(r_{m,t+1}\) is the market return for the subsequent period.

In theory, unless the market index used contains all of the assets in the economy, and these are weighted correctly, then there is the possibility that betas estimated will be distorted. Notwithstanding, in practice market indices containing partial portfolios of assets are used, such as the Standard and Poor’s 500 (Merrill Lynch) or the New York Stock Exchange composite market index (Value Line). If the aim is to calculate relative betas, then the choice of market index ceases to be relevant. In general, the ranking of betas is less sensitive to differences in the period examined and the interval of return than the estimate of absolute betas is.

There is an effect which has been much studied by financial literature which refers to betas’ tendency to converge towards their mean (1.00). The basic idea is that the estimate for beta will be in part a function of the true beta, and in part a function of the sample error which has taken place. If a high estimated beta is calculated (ie. one which is above the average of one) then there is a greater probability that a positive sample error has occurred, while if the estimate for beta is low, then it is more likely that the sample error is negative. If this scenario is correct, then it is feasible that on average betas will tend to converge towards one over successive periods of time.

That is, high betas tend to drop over time, and low ones tend to rise. As a result of this trend, betas estimated using the above methods are not suitable proxies for the expected betas. This is why commercial publications, such as Value Line or Merrill Lynch, calculate the so-called adjusted beta, which is the best proxy of the expected beta:

Adjusted \(\beta = 1.00 + K (\beta - 1.00)\)

where \(K\) is an estimate (arising from historical data) of the extent to which estimated betas regress towards the average.

Merrill Lynch, for example, obtain betas by using a weighting for beta estimated at 66% (\(K=0.66\)). This method of correcting betas was suggested by Vasicek (1973). However, Vasicek was suggesting that the weighting of the adjustment should depend on the degree of uncertainty over the beta (the sample standard deviation of beta). The greater the sample deviation, the greater too will be the probability of significant differences between the value observed and the average (where beta is equal to one), due to the sample error, and the greater the adjustment ought to be (Elton & Gruber, 1991).
One problem related to the above is the instability of beta. Even in the cases of estimates which are corrected by the trend, the estimated betas tend to appear unstable, which is really a problem if the beta value is to be used for calculating the cost of capital (if the future beta is changed, the current cost of capital will not reflect the true opportunity cost of invested funds).

When a company is not quoted on the stock exchange, it is possible to calculate measurements of risk based on betas of comparable firms. One difficulty with this approach is due to the fact that although firms may be subject to the same “business risk”, they may have different ownership structures, as is explained below.

Betas measure two types of risk: the fundamental risk of the business and financial risk. However, in order to be able to carry out comparisons it is only necessary to consider fundamental risk, which is calculated through the unleveraged beta or the beta of the asset (\(\beta_a\)). If it is assumed that the beta of debt is zero (ie. all risk is taken on by the shareholders), then

\[
\beta_a = \beta / [1+(1-t)D/KP] \quad (2).
\]

Simplifying this,

\[
\beta = \beta_a \cdot [1+(1-t)D/KP] \quad (3).
\]

An increase in financial leverage, ceteris paribus, will increase the beta of the firm’s equity. Intuition suggests that interest payments (as a result of debt increases) increase the variability of net revenue: greater leverage will increase net income in times of prosperity and reduce it in times of economic hardship.

Synthesising this information, we can see that if one wants to calculate the beta for a firm (A) from the beta of a similar firm (B), one must first calculate the beta of firm B including the business and financial risks. This beta must be deleveraged using formula (2), and then releveraged using formula (3). We should make it clear that leverage using formula (2) corresponds to firm A, while that using formula (3) corresponds to firm B.

To the extent that beta reflects the risk associated with a firm, what should one expect of the betas of regulated firms? With the exception of the case of those firms which are under a cost-of-service regime, there is no reason to suppose that the beta value should necessarily be any lower than one. In the first place, in a regulatory system where maximum prices are fixed, tariff reviews take place very rarely. During the period in which the fixed price is in operation, with specific exceptions when the transfer of extraordinary costs is allowed, prices can only be adjusted through RPI. The impossibility of responding (via tariff variations) to economic changes will increase the volatility of profits and therefore also the beta associated with the firm. Secondly, in regulated firms there are fixed costs which increase the risk of equity. Many privatised companies have to fulfill obligations in terms of quality and supply, which as a result gives them limited flexibility over their future manoeuvres. In contrast with this, non-regulated firms can postpone capital expenditure and reduce the quality or provision of the service if they so wish.
III. Incentives and Strategic Behaviour

The components of the cost of capital can be differentiated into those over which the firm has control (and which can therefore feasibly be influenced by strategic behaviour on the part of the firm) and those over which the firm exercises no direct control. These latter include the opportunity cost of equity, while the former includes the cost of debt and financial leverage.

The capital structure is controlled by the firm and the firm can therefore decide upon a capital structure which will allow them a greater return on their own capital invested. In the case of the cost of debt, the effort made to be able to go into debt at low rates becomes the key issue for the firms.

The cost of debt

The first question to ask is how the decision to use the marginal or average cost affects the firm’s incentives. If the cost of debt depends positively on the level of leverage, as can happen in reality, recognising a firm’s cost of debt equal to its marginal cost implies that the forward-looking firm will always have incentives to increase its financial leverage, as this would make the real cost of debt (the mean cost) lower than the marginal cost, and equity would get higher returns than those determined by the cost of equity.

However, although it is true that the forward-looking firm obtains profits when it becomes over-indebted, it is also true that it will have incentives to lower its cost of debt during the period of tariff review (given that during this period the cost of capital is fixed), although it will also have incentives for that rate to increase immediately before the review. This kind of perverse incentive is less powerful if the cost of capital is calculated with the mean cost, since the effect of marginal debt on mean cost is insignificant. In other words, use of the marginal cost would imply that the regulator “has no memory”, while use of the mean cost implies that the regulator “never forgets”.

On the other hand, the use of mean cost can reduce the firm’s incentives to carry out expansion, just in case this would have to be financed through new debts. If the marginal cost of debt is higher than the mean cost, then the firm would be losing profit on its own invested capital every time it does not recognise future costs linked to expansion as the mean cost of debt. In this way, from the incentive point of view, the most suitable would be to use the mean cost of debt, including in these costs those associated to the firm’s future investments.

Furthermore, in the case of regulated firms, and for reasons of efficiency, it would be possible to argue for the convenience of calculating $r_d$ by calculating an average between the firm’s cost of debt and the mean market cost of debt. In this way, one would be dividing the potential benefits of efficiency between the firm and the consumers, and these benefits would be reflected in the firm’s having a rate of debt lower than the average. If the effective rate of debt were used (and if this were lower than the market average), then all benefits arising from efficient management of the firm would be appropriated by consumers, while if the market average rate were used then all benefits would be taken by the firm (as it would be recognising a cost of capital higher than the actual one).
However, in terms of the firm’s incentives, and assuming that the costs of the effort necessary to lower the rate of debt are nil, then any rate fixed for the whole of the period of tariff review will offer the firm incentives to lower it, since whenever the firm manages increases in efficiency in the handling of its debt it will be in a position to appropriate any profits for itself, at least up until the moment of the subsequent revision. However the costs of the effort required are not nil, and therefore the company will only make the effort to become more efficient when the present value of income (due to being able to become indebted at lower rates) is greater than the costs of the effort necessary to reduce these rates.

That is, the magnitude of the firm’s incentive to reduce its cost of debt will depend on the period for which the firm will be able to appropriate profits derived for this reduction. In the case where they average their own costs with the market averages (and there is no reason why the weighting should be a mean), the firm can ensure that not all of the profits resulting from the efforts they have made will be taken away with the next tariff review, and therefore the probability that the firm will seek to lower its cost of debt will be greater.

In sum, averaging the costs of debt of the market with those of the firm means that regulated firms have incentives to make an effort to lower their costs of debt (which, through lower rates, will benefit consumers), although for this it is necessary that (since effort costs do exist) firms are allowed to appropriate a part of the profits gained from a lowered cost of debt.

*Capital structure*

Concerning the determining of the firm’s optimal capital structure, there are two areas where literature on the subject tends to focus its attention: taxes and the cost of bankruptcy. From the tax point of view, debt is valuable since interest payments are tax-deductible. The counterpart to this is that an increase in the level of leverage increases the probability that a firm goes bankrupt, which has its costs. The point at which a balance is found for this trade-off determines the firm’s optimal level of leverage.

This literature begins with the work of Modigliani and Miller (1958). In its basic model, with perfect capital markets and no taxes or bankruptcy costs, the cost of capital was not affected by the firm’s ownership structure. In this way the capital structure became irrelevant. Including only taxes on corporations in the model produced a diametrically opposite result. Given that interest payments can be tax-deductible, the optimal ownership structure is one of 100% debt. This extreme and unrealistic result disappears when, besides including corporation taxes, one also includes costs associated with bankruptcy or personal taxes. In this case the optimal capital structure will depend on the rates corresponding to the respective taxes. Furthermore, independently of taxes, the extreme case cannot be preferred by share-holders or by creditors, as both would see an increase in the probability of defaulting.

From a regulatory point of view, if only corporate taxes existed, Modigliani and Miller’s analysis implies that regulators could pass tax savings on to consumers by lowering the maximum price the firm is allowed to set. Thus regulators could provide an incentive to firms’ debt. However, using these results can be dangerous, due to the fact that they do not take into account the harmful effects of the increase in the firm’s financial leverage (Scott, 1976). In any
case, both share-holders and regulators will want the firm to be in some debt, since this produces a situation in which the banking system acts as “co-regulator”.

Including costs of bankruptcy, the greater the firm’s leverage, the greater too will be the probability that the firm will incur bankruptcy costs. Analysing this, this is where there is a divergence between regulated and non-regulated firms. Although competitive firms will take bankruptcy costs into account when deciding their capital structure, consumers will not suffer any losses as the remaining firms would be able to service all demand. In the case of regulated monopolies, bankruptcy can have negative effects for consumers, in the event of there being no other companies that can cover demand instantly, or simply through an increase in tariffs due to the increased cost of capital the firm will be facing in the future. This implies that social costs are greater than private costs. The firm only takes private costs into account at the moment of deciding upon the capital structure, and therefore its optimum level of leverage will be greater than the level desired by the regulating authority, who does take account of social costs resulting from bankruptcy.

The effects of this divergence between social costs and private costs is exacerbated by the fact that the government subsidises the use of debt through the tax structure. This creates private profits derived from financing through debt which has no counterpart in terms of social gain, except where the regulator decides to transfer these benefits to consumers through decreases in tariffs. The simplest way of eliminating this difference between social and private gains is the setting of an upper limit to the total amount of debt a regulated firm can accumulate.

Synthesising this, we are faced with a problem of moral hazard, as the contract between the principal (the regulator) and the agent (the firm) offers the agent incentives to make choices which are not optimal for the principal.

IV. The Type of Regulation and the Associated Risk

The risk premium of the industry, reflected in the beta of the firm, will depend on the type of regulation the firm is subject to. Below we shall detail the most commonly used kinds of regulation:

Regulation through price-caps. Prices are fixed over a long period of time, while also determining the kind of price indexing during the period in which this regulation is in force. This kind of regulation offers incentives to the firm to be efficient, as a company can succeed in increasing its profit rate whenever it reduces its costs. However, this is achieved at the cost of increasing the firm’s exposure to risk: where there are no price adjustments, the company is exposed to any possible changes in costs, including those over which it has no control. This greater risk is reflected (through a higher beta) in an increased cost of capital. In practice, price-cap regulations require periodical revisions of prices in order to be able eventually to transfer to consumers the benefits derived from improvements in efficiency. Note that in these cases the regulator’s “memory” is crucial. If the regulator does not have a good “memory”, that is, it fixes tariffs based on costs at the moment of the tariff review) the firm has incentives to lower its costs for the period during which tariffs are fixed, and increase them (or overestimate them) for the period in which revision is to take place. This perverse incentive is lessened in cases where those
costs considered are the average costs for the whole period preceding the tariff reform. The disadvantage of this method is that if the firm reduced its costs regularly over said period, then it would be allowing itself, being forward-looking, to take possession of a part of the gains from efficiency. An alternative mechanism consists in using the minimum between the average costs of the period and the cost of the last period.

*Price-caps with pass-through.* This system allows variations in costs which are not controllable by the regulated firm to be passed through to prices. This reduces the risk faced by the firm, and therefore also the cost of capital. Incentives will not always be undermined whenever the costs which can be transferred are really outside the firm’s control. There is, however, the possibility that the cost pass-through system is used asymmetrically by the regulating body, thus exposing the firm to higher, instead of lower, risk. In this case a regulator would use its discretionary powers to appropriate the company’s profits when exogenous costs fall, but refusing to allow the increases in these costs to be transferred to consumers.

*Revenue-cap.* This system limits the total revenue of the regulated firm; it is suggested that it be implemented in cases where most costs are fixed. This reduces the risk in comparison with a price-cap regulation, as in the case of the latter the firm would be affected by fluctuations in demand (note that these fluctuations affect total income while leaving costs unchanged). It is worth emphasising that this system reduces risk without undermining incentives to reduce costs faced by the firm.

*Rate of return regulation.* In this case the investor faces very low risk, as any variation in demand or costs is adjusted via alterations to prices. One disadvantage of this system is that is does not provide an incentive to efficiency, while it promotes firms’ over-capitalisation (the Averch-Johnson effect). The idea behind this is that the mechanism for regulation through the rate of return distorts the choice of the mix of inputs, as profits are linked to the size of the capital stock. With a mechanism of regulation through price-caps, there is no direct relation between the purchase of a particular input and profits. There is therefore no reason to expect that the mechanism would intrinsically lead to an inefficiency in allocation.

The differences in the kind of regulation affect non-diversifiable risk (reflected in beta), with this being greater when regulation is carried out via price-caps and less when it is through rate of return. Likewise, there is an inverse relation between the degree of risk and the level of incentive faced by the firms. The following table shows values of beta for the asset (ie. corrected by the level of leverage of individual companies) by country and by sector.

**TABLE 2**

<table>
<thead>
<tr>
<th>Beta values of assets by country and by sector</th>
<th>Electricity</th>
<th>Gas</th>
<th>Energy</th>
<th>Water</th>
<th>Telecom.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Britain</td>
<td>0.60</td>
<td>0.84</td>
<td>-</td>
<td>0.67</td>
<td>0.87</td>
</tr>
<tr>
<td>USA</td>
<td>0.30</td>
<td>0.20</td>
<td>0.25</td>
<td>0.29</td>
<td>0.52**</td>
</tr>
<tr>
<td>European</td>
<td>0.41</td>
<td>0.49</td>
<td>0.46</td>
<td>0.42</td>
<td>0.70</td>
</tr>
<tr>
<td>Countries*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* selected</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>AT&amp;T not included</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Source: Alexander, Mayer y Weeds, 1996.*
Regulatory systems can be classified according to the incentives they produce: RPI-X and revenue-caps offer greater incentives, while European discretionary ones are at an intermediate level.

Table 3 shows a summary of the kinds of regulation used by each country and each sector, while in Table 4 we can see the beta values for assets by type of regulation and sector.

**TABLE 3**

Summary of regulatory systems in force in selected countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Electricity</th>
<th>Gas</th>
<th>Energy</th>
<th>Water</th>
<th>Telecom.</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>RoR</td>
<td>RoR</td>
<td>RoR</td>
<td>RoR</td>
<td>RoR**</td>
</tr>
</tbody>
</table>

*RoR means rate of return regulation

* selected

**AT&T is regulated through RPI-X


**TABLE 4**

Beta values of assets by type of regulation and sector

<table>
<thead>
<tr>
<th>Incentives</th>
<th>Electricity</th>
<th>Gas</th>
<th>Energy</th>
<th>Water</th>
<th>Telecom.</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>0.57</td>
<td>0.84</td>
<td>-</td>
<td>0.67</td>
<td>0.77</td>
</tr>
<tr>
<td>Medium</td>
<td>0.41</td>
<td>0.57</td>
<td>0.64</td>
<td>0.46</td>
<td>0.70</td>
</tr>
<tr>
<td>Low</td>
<td>0.35</td>
<td>0.20</td>
<td>0.25</td>
<td>0.29</td>
<td>0.47</td>
</tr>
</tbody>
</table>


These results, obtained from a significant number of cases, show a clear pattern of differentiation between the distinct sectors of public services and between alternative regulatory systems. Systems with low incentive power (rate of return regulation) coexist with low beta values, while those whose incentive power is high have significantly higher beta values. These aspects should be considered if one wants to estimate the beta of a regulated firm from the beta of comparable firms.

V. Regulatory Practice in Argentina

The expected rate of return on equity capital invested in firms based in Argentina should include a country risk premium which compensates investors for the greater risk undertaken.
Usual practice in this subject is to adapt the CAPM to include a risk associated with the country in which activities are developed (Visintini, 1998). Formally, this is

\[ r_{kp} = r_1 + \beta (r_m - r_1) + r_{country} \]

where \( r_{country} \) is the country risk premium. In theoretical terms the country risk is the spread between a bond issued by the national government and one of similar duration from the United States government (or that of a similar country). If both bonds are issued in the same currency, the differences in yield would only be default risk, while if the national bond is issued in domestic currency then the spread would also include the devaluation risk.

In order to calculate the country risk premium to be included in the formula for the cost of capital, it is important to emphasise that this rate should be estimated from the spread of existing bonds (or created ad hoc with the existing ones) which reproduce a series of net incomes similar to the firm’s.

That is, using the firms’ estimated flow of funds it is possible to calculate the average life of the firm, using the following formula:

\[ VU = \frac{1* FC_1 + 2* FC_2 + ... + n* FC_n}{FC_1 + FC_2 + ... + FC_n} \]

where \( VU \) is the average life, \( n \) the maturity of the firm (what is left of the concession) and \( FC_j \) the cash-flow of year \( j \). Once the average life of the firm has been calculated, the next step is to calculate the difference between the internal rates of return of a bond (or portfolio of bonds) issued by the United States government and for one (or a portfolio) issued by the national government, with both having the same average life as the firm.

However, the calculation of average life adds nominal weighted flows without recognising the value of money in time. This is why an alternative indicator which could be used is that of modified duration which incorporates the value of weighted flows but in terms of current value. However, although it is true that duration is the best measure of risk faced by the investor, the calculation of duration implies the use of the rate of discount. Since the original aim was, in the first place, to calculate precisely this rate (the cost of capital), we are faced with a problem of circularity.

It is important to emphasise the significance of the moment or period for which the spread is calculated, as this is enormously sensitive to circumstantial factors in the markets. For example, the spread between the FRB (bond in dollars issued by the Argentine government) and the 10-year U.S.Treasury bond had values averaging 493 basic points for the first quarter of 1994, increasing consistently through that year to reach a peak of 1496 basic points as the average for the first quarter of 1995, when the effects of the Mexican crisis were most in evidence in the region. Towards the end of that year, the country risk had dropped (1082 basic points for the last quarter of the year). 1996 saw a sustained reduction in the country risk, reaching values of 518 basic points for the fourth quarter of the year. The two subsequent years saw the country risk unable to stabilise. At present (January 1999) the spread between the two bonds examined is of the order of 1212 basic points, as it has been severely affected by the news of the Brazilian
devaluation. Taking the bond index of Merchant Bankers & Associates, which includes all papers of Argentina’s internal and external debt, the spread weighted with respect to the United States Treasury Bonds with equal terms has reached levels above 1200 basic points. The extreme volatility is reflected in the following piece of data: the same spread, one week earlier, was at slightly above 710 basic points.

Figure 1 shows the relation between average life and IRR of bonds in dollars issued by the Argentine government, at the middle of January 1999.

Once a firm’s average life has been estimated, it is possible to estimate the IRR of the two portfolios of bonds (those issued by the national government, and those issued by the government of the United States) by running a regression. The regression can be a linear one, or quadratic or logarithmic, to name only those most commonly used. The differences between the IRRs estimated with different models are not insignificant. For the data shown in Figure 1, the estimates produce the following results:

**Linear regression**... \[ \text{TIR} = 11.99063 (10.92) + 0.001147 (1.75) \text{Average Life}; R^2 = 0.13. \]

**Quadratic regression**... \[ \text{TIR} = 11.79760 (7.16) + 0.001553 (0.59) \text{Average Life} - 1.07 \times 10^{-7} (-0.16) (\text{Average Life})^2; R^2 = 0.14. \]

**Logarithmic regression**... \[ \ln \text{TIR} = 1.724489 (4.22) + 0.124967 (2.05) \ln(\text{Average Life}); R^2 = 0.18. \]

Respective statistics “t” are given in brackets, and \( \ln \) represents the natural logarithm. For an average life of 10 years (3650 days), the estimated IRR would be 16.18%, 16.04% and 15.63%, for linear, quadratic and logarithmic models, respectively. For an average life of 20 years, in turn, estimated IRRs would be 20.36%, 17.43% and 17.04%, again for linear, quadratic and logarithmic models, respectively.

In his work for the sectors of the transport and distribution of gas in Argentina, Visintini uses a log-specification.

**Figure 1**
Relation between IRR and average life

*Type of Regulation.* All firms in Argentina are regulated with a maximum price mechanism, with RPI-X+K.

*Capital Structure.* Below we provide a table with levels of leverage for regulated firms quoted on the Buenos Aires Stock Exchange in the middle of January 1999. Leverage was calculated as the relation between the value of the debt and the total value of assets, according to the latest General Balance recorded by the firms.

**TABLE 5**

<table>
<thead>
<tr>
<th>Sector and firm</th>
<th>Leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas distribution</td>
<td>0.44</td>
</tr>
<tr>
<td><em>Gas Natural BAN</em></td>
<td></td>
</tr>
<tr>
<td>Gas distribution</td>
<td>0.45</td>
</tr>
<tr>
<td><em>Metrogas</em></td>
<td></td>
</tr>
<tr>
<td>Gas transport</td>
<td>0.44</td>
</tr>
<tr>
<td><em>Transportadora de Gas del Sur</em></td>
<td></td>
</tr>
<tr>
<td>Electricity distribution</td>
<td>0.42</td>
</tr>
<tr>
<td><em>Edenor</em></td>
<td></td>
</tr>
<tr>
<td>Electricity distribution</td>
<td>0.25</td>
</tr>
<tr>
<td><em>Edesur</em></td>
<td></td>
</tr>
<tr>
<td>Telecom</td>
<td>0.52</td>
</tr>
<tr>
<td><em>Telefónica de Argentina</em></td>
<td></td>
</tr>
<tr>
<td>Telecom</td>
<td>0.56</td>
</tr>
</tbody>
</table>

One aspect which it is important to bear in mind when calculating the beta of a regulated firm in Argentina is the market against which it is to be measured. Given that the S&P 500 index is used as the market indicator, the correct procedure would be to calculate the beta as the relation between, on the one hand, the covariance of the return of the firm under examination and the yield of the S&P 500 index, and on the other the variance of the return of the S&P 500 index. Formally, this is

$$\beta_j = \frac{\sigma_{jm}}{\sigma_m^2}$$

where $\beta_j$ is the beta of equity capital of firm $j$, $\sigma_{jm}$ is the covariance between the return of the firm under consideration and the yield of the S&P 500 index, and $\sigma_m^2$ is the variance of the yield of the S&P 500 index.

There follow some estimates for the beta coefficients, for firms quoted on the Buenos Aires Stock Exchange. These coefficients were calculated considering two possible cases: taking the local index (MERVAL) or the S&P 500 index as the market.
TABLE 6
Weekly Betas (without adjustments)

<table>
<thead>
<tr>
<th>Firm</th>
<th>Beta (MERVAL)</th>
<th>Beta (S&amp;P 500)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tranportadora de Gas del Sur</td>
<td>0.595</td>
<td>0.853</td>
</tr>
<tr>
<td>13/5/94-21/12/98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Telefónica de Argentina</td>
<td>0.859</td>
<td>1.229</td>
</tr>
<tr>
<td>3/1/92-21/12/98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Telecom</td>
<td>0.876</td>
<td>1.308</td>
</tr>
<tr>
<td>30/3/92-21/12/98</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Quinquennial tariff review in the Gas Sector

The WACC was used to calculate the cost of capital which was taken as the basis for establishing tariffs, in force from the quinquennial tariff review of the First Quarter of 1998. The cost of equity capital was calculated using the modified form of the CAPM presented above.

According to the content of the report (Visintini, 1998) “The value of debt and equity should be expressed at market value, and for those firms without instruments of debt issued on the market and which are not quoted on the stock exchange one should use the information available on the balance-sheets.”

The risk-free rate was calculated from the internal rate of return of the Treasury Bond of the U.S. Treasury, with an average duration similar to that of the companies analysed.

The beta coefficient could not be estimated directly, as at the time the work was carried out there were only two companies being quoted on the Stock Exchange (Transportadora de Gas del Sur and Metrogas), with very low levels of liquidity of the papers of the latter. Furthermore, the statistical series was not extensive enough to produce reliable results. The beta was therefore calculated from the beta of comparable firms. First of all, the beta for firms in the sector in the U.S. were calculated, and then this was corrected in order to take into account the differences in the type of regulation. This difference, estimated from the beta of firms in the telecommunications industry in the U.S. (where some firms are regulated by rate of return and others by price-caps) was added to the original beta.

The cost of debt of companies in the sector was calculated as the sum of the country risk with the risk-free rate of the U.S. Treasury for the average life of the companies. That is, the firms’ cost of debt was not taken into account.

The country risk was calculated from the average life of a group of bonds corresponding to the Euronotes market.
The average risk premium of the market was calculated using the geometric means (the Standard and Poor’s 500 index), with the argument that it is these that reflect the true profit obtained over a given period of time. However, as has already been reported, the literature is not conclusive on this matter.

The importance of this point is not only theoretical but empirical. Depending on the period chosen for use by the author, the minimum hypothesis of the average risk premium of the market could increase from 3.03% to 3.60%, while the maximum hypothesis would go from 6.45% to 8.29%. The impact on the total cost of capital would be far from worthless. For distributors a minimum hypothesis of 13.09% would rise to 13.30%, and that of a maximum hypothesis from 15.38% to 16.40%. In that case there is a difference of over 1%. For transporters, in turn, a minimum hypothesis of 11.96% would rise to one of 12.20%, and a maximum hypothesis from 13.48% to 14.20%.

According to ENARGAS’s final resolution, “the rates to be used in tasks declared will be 13.1% for the licensed activity of distribution of gas by networks, and 11.3% for the licensed activity of transportation of gas by pipelines”.

VI. An Investment Model of a Regulated Firm

In a 1971 work, Jorgenson and Handel maintained that “for the regulated industry the theory of optimal accumulation of capital is the same as for non-regulated industries, except for the imposition of an additional restriction on the maximisation of present value. Given the product price established by the regulatory agency, the quantity of the product is determined by public consumption.”

This view of the problem can be significantly amplified in the light of current knowledge about eventual increases in efficiency and the implicit trade-offs between the choice of tariff systems, the cost of capital and the service obligations in terms of quantity and quality.

In order better to understand this evaluation we are shown a process of maximisation of the present value of a firm which chooses an investment plan between today and some time T in the future, and which is subject to the control of a regulatory agency which defines aims, restrictions and incentives respecting the firm’s sustainability.

In general terms, it is assumed that the firm is risk-neutral even though, as will be seen below, this result will in fact have to be relaxed slightly to make it possible to discuss aspects of cost of capital and sustainability.

The main aim is to appreciate how the regulatory system and the clauses in the concession contract determine investment policy. Thinking in the context of a process of restructuring and privatisation such as that which developed in Argentina in the last few years, the problem faced by policy makers lies in defining this group of clauses in the concession contract in such a way as to maximise social welfare. The duration of the contract, the tariff regime, the investment aims, the way in which this investment is financed, and the minimum standards of quality are among the tools available at the moment at which the transformation of a sector is defined.
The choice of instruments should be in response to aims of sustainability, productive efficiency, assignment efficiency and equity, in such a way as to maximise social welfare in the long term. One point which is central to this problem is the existence of trade-offs between these aims, in that an improvement in one can often only be achieved to the detriment of another.

There follow some examples which help to illustrate this regulatory dilemma:

- An increase in the proportion of investment directly paid by consumers (µ) together with a reduction in the investment to be recovered through tariffs (Po) results in increased equilibrium investment (K*) without affecting the firm’s sustainability. However the effect of this policy on wellbeing will depend on consumers’ income elasticity (affected by the increase in the infrastructure charge) and price elasticity.

- Incentives to invest can be improved by extending the duration of the concession (T), which will lead to an intergenerational trade-off, as by extending the duration a part of the current investment costs are being transferred to future generations. A similar effect is associated with increasing the compensation paid upon completion of a concession.

- Extending the duration of the contract (T) improves incentives to invest, but at the same time it also increases the cost of capital, increasing the risk associated with a contract which is necessarily less complete.

- A tariff regime with high incentive power induces productive efficiency but reduces coverage (K*) and increases the cost of capital (r).

The existence of these trade-offs between aims is central to the regulatory problem, both regarding the design of the original contract and the way in which it is administrated over its duration by the regulatory agency. A clear understanding of these various elements and the way in which they interact is therefore absolutely essential.

The model

We begin with a traditional model of investment decisions with adjustment costs.

Without regulation, and even with the price (or tariff) fixed endogenously (by pressure of competition) the firm determines its investment policy to maximise its present value (PV):

\[
VP = \int_0^T \left[ p(\alpha)q - c(q, \alpha) - \phi I - \xi T^3 \right] e^{-rt} dt,
\]

subject to

(4) \( K = I - \delta K \)
(5) \( q \leq \gamma K \)
In these expressions, $q$ is the quantity produced and sold independently of prices, but which requires a minimum availability of capacity per unit of the product given by equation (5), while $K$ develops according to (4), where $I$ is the total of gross physical investments (of the index of capital goods) and $\delta$ is the rate of depreciation.

In the expression for $PV$, $T$ is the time the firm intends to remain in the market (determined by the firm itself), $p(\alpha)$ is the product’s unit price which is estimated based on a quality parameter, so that:

$$p'(\alpha) \geq 0,$$

according to the evaluation of consumers. Here quality is achieved through higher operating costs, that is, $c'_\alpha$, although eventually a substitution between $K$ and operating costs could be possible.

The cost of capital is given by $r$, which for the moment we can consider constant, but which, on the whole, with certain assumptions, is composed of a number of elements:

(6)  $r = r_1 + \beta (r_m - r_1) + \text{primadeval} + \text{primadefault}$

That is, the cost of capital is the sum of the risk-free rate, $r_1$, and the proportion of the market’s risk premium $(r_m - r_1)$ which determines the degree of co-participation which this industry has with the market, given by $\beta$, plus the premiums of devaluation and default if there is an ex-ante admission of the free mobility of capitals.

There are two types of investment costs: current price (taking $r$ to be nominal) of the investment goods $\phi$ times the investment index $I$ gives the estimated expenditure on investment goods if supply is completely elastic; however it is accepted that if (new and repositioning) investment plans are over-ambitious (working too quickly) it may be the case that costs in the $\xi I^2$ term will increase substantially, where $\xi$ is a positive parameter.

Allowing for the fact that the firm does not face indivisibilities of $K$ which oblige it to bring iddle capacity along with it, condition (5) will be fulfilled as an equation:

(5') $q = \gamma K$

That is, $\gamma$ is the mean physical productivity of capital.

This hypothesis is not as naive as it appears. Many regulated sectors have marked indivisibilities of the capital stock, which means that they pay high costs of capital on assets immobilised by poor prediction of demand or because regulation obliges them to satisfy this demand immediately, without expecting the thresholds of economic viability to be exceeded.

In these conditions the Hamiltonian, the increment in the total value of the firm’s assets, is given by:

(7)  $[p(\alpha)\gamma K - c(\gamma K, \alpha) - \phi I - \xi I^2] e^{-\tau} + \lambda (I - \delta K)$
where \( \lambda \) is the shadow price of capital.

Maximum conditions therefore require that:

\[
(8) \quad H_d = [p'(\alpha)\gamma K - c'_{\alpha}] e^{-rt} = 0,
\]

\[
(9) \quad H_I = -(\phi + 2\xi I) e^{-rt} + \lambda = 0,
\]

\[
(10) \quad H_k + \lambda e^{-rt} [p(\alpha)\gamma - c'_{\gamma}] - \lambda \delta + \lambda = 0,
\]

plus transversality conditions:

\[
(11) \quad H(T) = 0
\]

which shows that at the moment of leaving the industry the firm does not see future increases to the value of its financial and physical assets, and:

\[
(12) \quad \lambda(T)K(T) = 0,
\]

\[
\lambda(T) \geq 0
\]

which shows that unused capital is not abandoned if its shadow price is positive.

When quality does not matter for the price a firm sets on a product, that is, \( p'(\alpha) = 0 \), one would expect \( \alpha \) in (8) to be zero.

On the other hand, equations (9) and (10) determine the basic equation of the evolution of capital which maximises Present Value:

\[
I = (r + \delta)(\phi/2\xi + I) - \phi/2\xi - [p(\alpha) - c'_{\gamma}](\gamma/2\xi)
\]

(13)

so that the steady state capital stock is given by (when \( I = I_0 = 0 \)):

\[
[r + \delta - \phi/\phi]\phi + \gamma c'_{\gamma} = p\gamma
\]

(14)

Figure 2 shows a representation of the firm’s investment and growth process, making it possible to better appreciate the way in which this process is affected by the model’s parameters.

In a case where the market is always profitable, once \( \alpha^* \) has been chosen, a firm will strive to reach \( K^* \), and there is only one way for doing this, that is, along path SS; given an initial capital of \( K_0 \), the firm defines an initial investment as \( I_0 \). The effect of the adjustment costs \( \xi \) is to change the tangent of SS and modify the speed of convergence to \( K^* \) (without affecting this). Path AA is the one followed by firms with greater adjustment costs: initial investment, I, would be smaller.
**Figure 2**

On paths SS and AA the present value of the firm is being maximised; a path like BB would only be chosen when the firm had to reach $K_1$ in an exogenously specified period $T_b$ (which of course requires an increase in the level of investment in order to proceed faster).

Figure 3 shows the effect of a change in the cost of capital (an increase of $r$), which reduces the capital desired in the long term (but might also modify the speed of convergence, though here we will assume that this is not the case).

**Figure 3**

In the case of a firm subject to regulation, the maximisation programme takes place under certain additional conditions. One must bear in mind that privatisation has certainly helped to increase the value of $\gamma$ (mean physical productivity of capital) and to reduce operational costs by incorporating new technical and organisational practices (such as new internal incentive structures).

One should, then, consider that a firm takes into account the information on the concession contract and on the tariff regime (for example) when calculating its investment policy.

One can assume that the firm’s operational costs can be broken up into two components: those which the regulator considers able to be manipulated, and those the regulator considers exogenous, and which it allows to be passed on to the tariff almost entirely without anything being left over.

$$C = \rho c(q, \alpha, E) + (1 - \rho) c(q, \alpha)$$

where $\rho$ is the manipulatable proportion and $(1-\rho)$ that which is not manipulatable. $E$ represents a parameter of effort in the reduction of costs which the firm can undertake, (managerial, relating to consultancy, for example). In order to simplify this, we will assume that the non-manipulatable part can be expressed linearly as a function of $q$ (i.e. $(1-\rho)c(q)$, and does not depend on $\alpha$.

Furthermore the regulator can acknowledge a proportion of costs $\mu$ of investment in tariffs, especially if it superimposes a condition of minimum investment (per period) or aims to achieve coverage objectives. In Argentina, mechanisms of this type are used, such as in the case of water where the original concession contract proposed a system in which new consumers had to pay an infrastructure charge which would cover costs of investments linked to their supply. This system showed itself to have serious problems, as the infrastructure charge showed high rates of default over the first few years, leading ultimately to its substitution with charges to all associated consumers. Returning to the model,
In this case the expected tariff takes the form
\[ p = p_0 + (1 - \rho)c + \mu(\phi I + \xi I^2) / q \]
That is, the firm recovers a proportion \((1-\rho)\) of its operational costs and a percentage \(\mu\) of its expenditure on investment.

A priori, as it has a partial pass-through system, the cost of capital is reduced the lower \(\rho\) is, and the greater \(\mu\), given that it changes (reduces) the \(\beta\) of investment. The duration of the programme (T) also alters the cost of capital, since the longer the contract the more incomplete it is.

Synthesising this information, it seems that we can expect the cost of capital to become a function of the coefficient of incentive power, which here is \(\rho\), of the proportion of the cost of shared investment \(\mu\), and of the contract’s duration.

\[ r = r(r_i, r_m, \rho, \mu, T) \]
whose properties include
\[ r_\rho > 0, \quad r_\mu < 0, \quad r_T > 0 \]

A corollary of this is the fact that the cost of capital becomes a variable determined in the initial contract. Thus, for example, a very strict price-cap regime which increases \(\rho\) will determine a higher \(\beta\) and will affect the discount factor of the optimisation programme; the firm will require greater compensation for higher risk incurred, and will reduce long-term capital \(K^*\), lowering the coverage intended for it.

The programme of optimisation of the expected present value is now given by:
\[
VP^* = \max_0^T \left[ p_0 q - \rho c(\alpha, q, E) - E - (1 - \mu)(\phi I + \xi I^2) \right] e^{-r(\rho, \mu, T) t} dt +
+ e^{-r(\rho, \mu, T) T} R\left[ (K(T) - K_0)(1 - \mu) \right] - K_0 \phi_0
\]
subject to conditions of:
- Investment rate
\[ I \geq I \]
- Quality
\[ \alpha \geq \alpha \]
- Duration of contract
\[ T = T \]
-Limits on production

\[ q \geq q \text{ or rather } K(t) \geq K(t) \ \forall t \]

In the expression for present value, the term

\[ + e^{-rt} R\{[K(T) - K_0](1 - \mu)\} \]

is the residual value of the firm which it receives as compensation at the end of the concession. One should notice that this depends on the final capital stock advanced with equity funds, and that the initial capital advanced (assuming that what was paid for assets was in relation to Ko) is left over from the PV. For a privatised company this is the value of continuing the process in an optimal way. We will assume that this residual value is given by

\[ R\phi[K(T) - K_0](1 - \mu) e^{-rt} \]

where R is the proportion of Residual Value which the market acknowledges (in an infinite series, \( R = R_0/r \)).

We also notice that in this case the condition of Minimum Quality is necessary for \( \alpha \) to be greater than zero, and that there is an assumption being made that demand does not respond to prices but rather evolves autonomously. This restriction is not necessary for \( E \), which will be determined by the firm in order to minimise costs (-\( \rho C' = 1 \)).

This also implies that the firm cannot anticipate a negative maximum present value if it means to participate in the transaction (sustainability condition).

Figure 4 shows the effect of imposing certain conditions on the investment programme and the dilemmas which this implies.

An increase in \( \rho \), lower coverage of operational costs, means that the long-term capital stock desired will be reduced (from A to B), and that investment will fall (by increasing operational costs and costs of capital).

**Figure 4**

This can be compensated for by increasing \( p_o \), or ultimately by extending the duration of the concession.

Figure 5 shows what happens when the duration of the concession is give by T and the firm fails to recover residual capital.

The optimal policy would then be not to leave a valuable residual capital stock in place. They would therefore follow path TT, reaching a final capital of zero at the end of the concession.
For the final stock to be $K^*$ in a finite period requires the imposition of a restriction of minimum final capital, of investment rate, or the defining of a duration for the concession which is sufficiently long for the firm to choose to approach $K^*$ of its own free will (with a 95 year concession, for example, this would be along a MM path).

The minimum investment rate lowers the present value of the firm and imposes goals set in advance which significantly increase the adjustment costs of investment. The pass-through of investment costs, given by $\mu$, compensates the firm by increasing the tangent of path $SS$; in principle it does not alter the capital desired in the long term except that the firm does not see it as valuable residual capital.

The calculations can be simplified slightly in order to provide some useful comparisons.

Let us assume that each operator considers that the concession process repeats itself ad infinitum, in such a way that it is possible to be left with a proportion $R$ of the residual value of the transaction.

We also make the following assumptions: quality and effort $E$ have been fixed exogenously and do not matter here; capital goods do not change their prices ($\phi = 0$), and tariffs are not being indexed.

In this case the investment equation would be:

\[ i = \left[ r(1-R) + \delta \right] \phi / 2\xi + (r + d) I - \gamma \left[ p_0 - \rho C'_q(\gamma K) \right]/(1-\mu) 2\xi \]

We can see that (13) is obtained by taking $R=\mu=0$ and $\rho=1$.

The steady state is therefore determined by:

\[ r(1-R) + \delta \phi (1-\mu) + \gamma C'_q \rho = \gamma p_0 \]

an equation which is equivalent to (14), if suitable values for the parameters are used.

What differences can be seen in the long-term capital stock of this firm, in terms of $\rho$, $\mu$ and $R$? Note that $r$ has been assumed to be a function of $\rho$ and $\mu$. So:

\[ \partial K^*/\partial \rho = -[\phi (1-\mu)(1-R) r'_{\rho}/\gamma + C'_q(\gamma K^*)]/\gamma \rho C''_q(\gamma K^*) < 0, \]

that is, if the power incentive coefficient increases, the long-term planned coverage falls. This is due to the increase in the Marginal Cost which the firm must face with its own resources, and to the fact that the cost of capital increases (the $\beta$ of the transaction increases). If a firm does not want $K^*$ to fall, the increase in $\rho$ (the decision variable) must be accompanied by a consistent increase in $p_0$.

Furthermore:
\[ \frac{\partial K^*}{\partial \mu} = -\{r(1-R)\phi(1-\mu) - [r(1-R) + \delta'']\}/\gamma C^* \rho > 0, \]
\[ \frac{\partial K^*}{\partial R} = -r\phi(1-\mu)/\gamma C^* \rho > 0 \]

showing that the long-term cover desired grows if one allows part of the costs of investment to be rolled-in on tariffs, and if the firm estimates a higher residual value.

**VII. Conclusions**

This work analyses the components of the formula for the cost of capital, discussing the incentives created for a regulated firm by each of the possibilities. The choice between the mean and marginal cost of debt, for example, affects the incentives the firm receives, making it possible for it to become indebted to a degree exceeding that which is socially desirable. However, as has been said above, not all debt is bad from the regulator’s point of view, since the existence of debt allows the banking sector to act as co-regulator.

The discussion of whether the means used should be geometric or arithmetic for the determination of the market’s average risk premium is also well covered in the literature, without any total consensus being reached. This work suggests that for such a logic-based problem, it would be appropriate to use arithmetic returns.

In regulatory practice in Argentina, the norm is to use an adapted version of the CAPM for calculating the opportunity cost of equity, in such a way as to bear the country risk in mind. Another important aspect relates to the calculation of the beta, as this must be calculated in relation to the same index used for estimating the market’s average risk premium (usually Standard and Poor’s 500).

Finally, we presented an investment model of a regulated firm. The aim of this model is to illustrate the way in which the incentive parameters (the tariff structure) affect the cost of capital faced by the firm, and through this, long-term capital in the sector.
Figure 2
Figure 3
Figure 5
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